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UNDERACTUATED SPACECRAFT CONTROL WITH DISTURBANCE COMPENSATION

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14. ABSTRACT
This research focuses on the development and application of three methods for the underactuated spacecraft attitude problem. The first method utilizes Solar Radiation Pressure (SRP) to restore linear controllability to a spacecraft with only two functional Reaction Wheels (RWs). The second method is concerned with the creation of drift sets that characterize initial conditions for a spacecraft with two RWs such that it can perform desired imaging missions given constraints and disturbances torques. The third method describes the use of a hybrid switching scheme for two external moments control and two internal moments control, as well as a Model Predictive Control (MPC) approach for two internal moments control. The advantage to these approaches are that they can be used for zero and nonzero angular momentum. Simulations are done on spacecraft nonlinear models to demonstrate each of the approaches.

15. SUBJECT TERMS
Underactuated, Attitude, Hybrid, MPC, Set-Optimization, Solar Radiation Pressure Torque

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1.0 SUMMARY

The failure of Reaction Wheels (RWs) in an array can impair the spacecraft's ability to perform imaging missions, during which a prescribed inertial orientation has to be achieved and maintained with high accuracy and precision. Even though spacecraft are equipped with four or more RWs for redundancy, the possibility of multiple RW failures does exist. Even though the spacecraft may still retain thruster-based attitude control capability, the attitude control accuracy that thrusters are able to attain and maintain is typically insufficient. In addition, the use of thrusters involves using fuel, which shortens the spacecraft's life. Hence with RW failures, the spacecraft becomes underactuated. Recent missions, such as Kepler and FUSE (Far Ultraviolet Spectroscopic Explorer), are exemplar of these situations [1,2]. The focus of this research was on the development of several approaches that could be applied to an underactuated spacecraft, satisfying high precision pointing requirements and compensating for disturbances. Three methods were explored:

1. Utilizing Solar Radiation Pressure (SRP) to restore linear controllability to a spacecraft with two RWs.

2. Creating a set-optimization technique that computes drift sets. These sets represent all initial orientations such that, if the spacecraft starts in the set, it would satisfy constraints up to a given time regardless of drift.

3. Developing hybrid and Model Predictive Controller (MPC) approaches for reorientation and disturbance compensation in underactuated spacecraft.

Multiple publications have been published, written, or are in the process of being written, and are related to the subject matter of this report. All such publications are listed in the Bibliography.

2.0 INTRODUCTION

In the case of an underactuated spacecraft with two internal torque actuators, the linearized dynamics are not controllable. In [3] the nonlinear system is said not to be accessible with two or fewer momentum wheels due to the conservation of angular momentum. In many studies, the angular momentum of the spacecraft system is assumed to be zero. In [4], under the zero angular momentum assumption, two nonlinear control techniques are presented. The first is a finite-time, discontinuous feedback law that induces a sequence of rotations. The second involves a diffeomorphic transformation to a simpler set of equations of motion that facilitates controller design. These control strategies are discontinuous, as the attitude of an underactuated spacecraft cannot be smoothly or continuously stabilized by any time-invariant feedback law [5,6,7]. Control techniques using the zero angular momentum assumption, however, are of limited practical use.

Under this assumption, a spacecraft with less than four RWs must spin down the RW speeds to zero, which as a consequence decreases accuracy and may further decrease their operational life.

3.0 METHODS, ASSUMPTIONS, AND PROCEDURES

3.1 Assumptions

For this work, a spacecraft configuration consisting of a spacecraft bus and four RWs is considered. During the spacecraft's mission, two of the RWs will fail, leaving two operational RWs about the first two principal axes of the spacecraft bus. The RWs are assumed to be thin and symmetric. The orientation of the spacecraft is characterized by 3-2-1 Euler Angles yaw (ψ), pitch (θ), and roll (φ), with the kinematics prescribed by

$$\dot{\Theta} = M(\Theta)\omega, \tag{1}$$

where Θ is a vector of Euler angles, ω is a vector of angular velocities, and $M(\Theta)$ is a matrix which is a function of Θ. Any three parameter parameterization of orientation has singularities. It is assumed that the spacecraft's mission does not require large attitude transients and never approaches singularity.

The dynamics of the spacecraft system are governed by Euler's equation,

$$(J + J_w W W^T)\dot{\omega} = -\omega \times \left((J + J_w W W^T)\omega + J_w W \nu\right) - J_w W \dot{\nu} + M_{ext}, \tag{2}$$

where J is inertia matrix of the spacecraft bus, J_w is the inertia of the RWs about the spin axis, W is the orientation matrix of the RW spin axes with respect to the spacecraft bus, ν is the vector of the reaction wheel speeds, and M_{ext} is the vector of external moments acting on the system. For all the methods discussed, the external moments are limited to control moments and SRP torques. In the development of the MPC approach, it will be assumed that there are no external moments acting on the spacecraft.

Specifically for the MPC approach, the 8-dimensional state vector for a spacecraft with two RWs can be reduced to a 5-dimensional set of equations due to the law of angular momentum conservation. These equations, called the reduced attitude equations, are

$$\dot{\Theta} = M(\Theta)(J + J_w W W^T)^{-1}(\mathcal{O}_{BG} H - J_w W \nu), \tag{3}$$

where $\mathcal{O}_{BG}$ is the direction cosine matrix from the inertial frame to the spacecraft frame and H is the inertial angular momentum vector.

The SRP model used in this work is based upon that of [8]. Since the distance from the spacecraft to the Sun is large, SRP is assumed to act evenly across the same panel. The total SRP exerted on a panel j is

$$P_j = -\alpha(\hat{u}_{nj} \cdot \hat{u}_s)(\hat{u}_{nj} + \beta_j \hat{u}_s), \tag{4}$$

whereα and β_j are constants which are functions of the distance from the Sun, solar flux, speed of light, and diffusion coefficient of the panel, $\hat{u}_{nj}$ is the normal to panel j pointed outward, and $\hat{u}_s$ is the unit vector from the center-of-mass to the Sun. The solar radiation torque exerted by panel j is given by

$$\tau_{jsrp} = (r_{jO} - r_{CO}) \times A_j P_j, \tag{5}$$

wherer_{jO} is the vector from the center-of-mass to the center of panel j, r_{CO} is the vector from the center-of-pressure to the center-of-mass, and A_j is the area of panel j. Since SRP is additive across all faces, the total SRP torque exerted on the spacecraft is the sum of the torques due to all individual panels.

3.2 Methods

3.2.1 Recovering Linear Controllability with Solar Radiation Pressure

For this approach, Solar Radiation Pressure (SRP) torques are included in the dynamics of the spacecraft model. For small deviation from inertial pointing, the linearized model can be used effectively for analysis and controller design. While the open-loop system is unstable, our results show that under appropriate assumptions, which are outlined in [9,10], the linear system becomes controllable. The assumptions include the solar radiation pressure center being different from the center-of-mass of the spacecraft.

With linear controllability regained, spacecraft stabilization can be achieved by conventional control schemes. A Linear Quadratic (LQ) approach was first applied due to its robustness, its optimal control properties, and its familiarity to aerospace engineers. A pole placement scheme was also used to improve convergence time, since with LQ the closed-loop response was relatively slow. Overall, by taking advantage of the change in the dynamics induced by SRP torques, two RWs are able to slowly correct the attitude errors over time.

After [10] was published, the press release [11] suggested that SRP is actually used (in an unspecified control scheme) to restore Kepler's mission controllability. The controllability analysis and results from this work, obtained independently of [11], are thus indirectly corroborated by experimental evidence in [11].

3.2.2 Disturbance Torque Counteraction with RWs and SRP Torques

In this part of the research we focused on characterizing the ability of the spacecraft to perform inertial pointing while compensating for disturbance torques caused by SRP. This is done by the computation of drift sets, which characterize the set of all initial conditions such that if the spacecraft starts in the set, it will achieve the desired inertial pointing given constraints on pointing accuracy, pointing time, angular velocity, RW speeds, and RW accelerations. Since small deviations are considered, a linear discrete-time model is derived, with the form

$$x(k+1) = Ax(k) + Bu(k) + w(k), \tag{6}$$

where x is the state (vector of attitude Euler angles errors and angular velocity errors), $u \in \bar{U}$ is the control (vector of two RW's torques) and $w \in \bar{W}$ represents the effects of unmeasured disturbances/uncertainties bounded by the set W. The constraints on the system are represented by n linear equality constraints on the state,

$$\bar{H}_i x(k) \leq \bar{h}_i i = 1, \dots, n. \tag{7}$$

These linear inequality constraints can be used to construct the setO_k, which is defined as

$$O_k = \{x_0 \in \mathbb{R}^6 | \bar{H}_i x(k) \leq \bar{h}_i, i = 1, \dots, n\}, \tag{8}$$

where $x_0 = x(0)$. The set $O_k$represents the set of all initial condition such that the spacecraft will satisfy constraints at time k. The intersection of all sets O_k up to a given discrete time instant p will give the set of all initial conditions such that the spacecraft will not violate constraints up to and including the time instant, p. In addition, these sets can be easily represented and visualized by polytopes. Future work will involve using these drift sets in order to construct a drift reaction controller.

3.2.3 Hybrid and Model Predictive Controller for Reorientation and Disturbance Compensation in Underactuated Spacecraft

The hybrid approach developed for the underactuated spacecraft problem is based on the feedback stabilization techniques developed in [12] and [13]. These techniques exploit the decomposition of the system variables into base variables and fiber variables. The base variables are stabilized to periodic motions with feedback, and the parameters of these periodic motions are adjusted at discrete time instants to induce the evolution of the fiber variables towards the desired equilibrium. In the case when the spacecraft is actuated by two external moments, the attitude and angular velocity of the two actuated axes are treated as base variables and the attitude and angular velocity in the third, uncontrolled axis are treated as fiber variables. In the case when the spacecraft is actuated by two internal moments provided by RWs, the base variables are the same as in the two external moment case, but the only fiber variable considered is the attitude about the uncontrolled axis. When the system was controlled by two external moments, the system could reject SRP.

When internal moment control was used, a periodic reset scheme was implemented for a controlled oscillation about the uncontrollable axis.

For the MPC approach, the reduced attitude equations were used, and hence it was assumed there were no external moments acting on the system. It was proven that for any nonzero angular momentum, the reduced attitude equations are linearly controllable. An MPC control law can then be applied to stabilize the desired equilibrium as long as the angular momentum vector expressed in the spacecraft bus fixed frame is in the range of the matrix W. If the angular momentum is not in the range, then under suitable conditions, the spacecraft will converge to a nonzero equilibrium.

Both the hybrid and MPC method above work for nonzero angular momentum. The topic of underactuated spacecraft control when the angular momentum is non-zero has not been studied extensively in the literature, but is of great importance. By creating controllers that include and utilize nonzero angular momentum, the operational life of the RWs can be extended. Studying nonzero angular momentum also increases the scope of the spacecraft underactuated problem, as zero angular momentum is a limiting case.

For zero angular momentum, the system is linearly uncontrollable. In fact, the attitude problem with two RWs and zero angular momentum cannot be stabilized through any smooth time-invariant feedback law. This is a consequence of Brockett's theorem in continuous time [5], which can be extended to discrete time [14]. However, nonlinear MPC can result in a discontinuous control trajectory. Therefore, a nonlinear solver will be applied in future work.

4.0 RESULTS and DISCUSSIONS

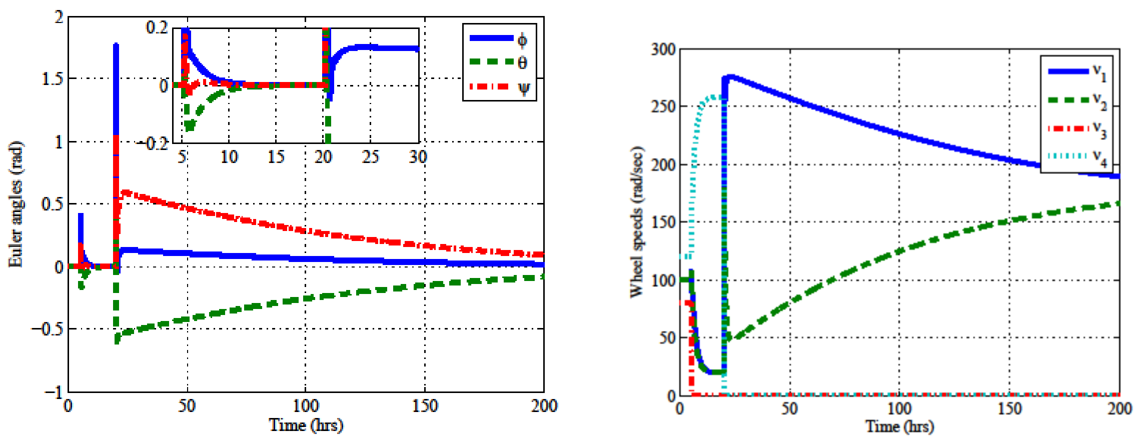
The simulations presented use the spacecraft parameters listed in Table 1. In Table 1, $\Phi_{sun}$is the solar flux, C_{diff} is the diffusion coefficient, assumed to be the same for all panels, and c is the speed of light. It is assumed that the two RWs remaining after failure are those two whose spin axis orientation correspond to the first two columns of W, (i.e. those aligned with two of the three principal moments of inertia).

Table 1. Spacecraft Parameters

Parameter	Units	Value
J	kg m^2	diag(430, 1210, 1300)
J_w	kg m^2	0.043
A_x	m^2	12.5
A_y	m^2	10
A_z	m^2	5
W	-	$\begin{bmatrix} 1 & 0 & 0 & 1/\sqrt{3} \\ 0 & 1 & 0 & 1/\sqrt{3} \\ 0 & 0 & 1 & 1/\sqrt{3} \end{bmatrix}$
Φ_{sun}	W/m^2	1367
C_{diff}	-	0.2
c	m/sec	299792458.0

4.1 Recovering Linear Controllability with Solar Radiation Pressure

The following simulation illustrate the case when RW 3 fails first, followed by RW 4. The center-of-pressure is offset from the center-of-mass by 0.5 meters. The responses are shown in Figure 1. There is a much larger disturbance to orientation when the second wheel fails. The controller manages the first wheel failure quickly, then reconfigures and handles the second wheel failure over a longer period of time coordinating two operational RWs in presence of SRP.



(a) **(b)**

Figure 1. (a) Euler Angles and (b) RW Speeds for When 2 RWs Fail and SRP is Used to Stabilize the Spacecraft

4.2 Disturbance Torque Counteraction with RWs and SRP Torques

In many spacecraft missions, it is desirable to perform rest-to-rest maneuvers. By setting the angular velocity entries of x_0 to zero, the linear equalities become functions of only orientation. Therefore the drift sets can be easily visualized. See Figures 2 and 3. For these drift sets, the limits on spacecraft orientation and angular velocity are 1 arc second and 0.0052 rad/sec respectively. Since an linear quadratic regulator controller is used, the RW accelerations can be directly obtained from the orientation and angular velocity. Therefore, a limit of 10 rad/sec^2 on RW acceleration is imposed. In both figures, the red box represents the constraints on pointing while the blue box is the set of all initial conditions the spacecraft can start and not violate constraints after k time steps. As k increases, the blue box will decrease and either converge to a set, as in Figure 2, or vanish if constraints are too stringent, as in Figure 3.

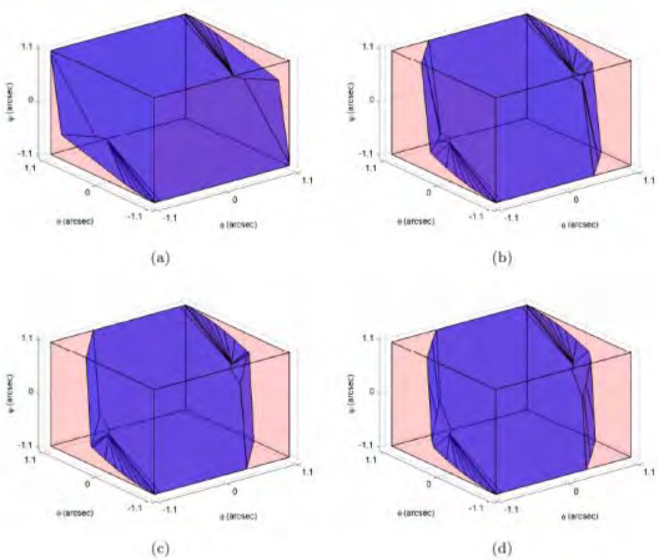


Figure 2. Drift Sets Calculation Over (a) 10 min (b) 20 min (c) 30 min (d) 40 min When the Controller Can Fully Compensate for SRP Torques

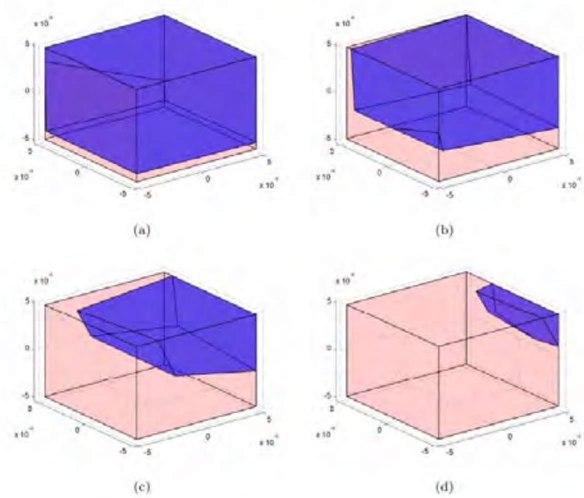
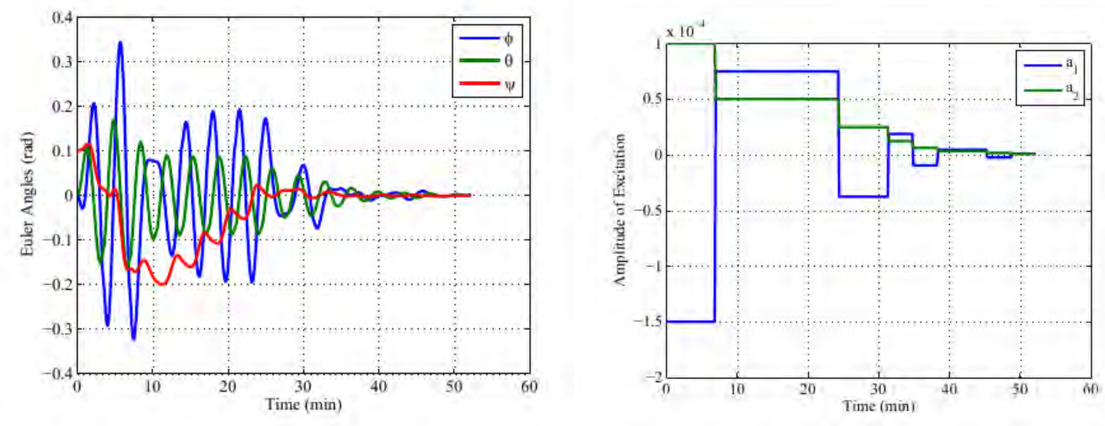


Figure 3. Drift Sets Calculation Over (a) 2 min (b) 4 min (c) 6 min (d) 8 min When the Controller Cannot Fully Compensate for SRP Torques

4.3. Hybrid and Model Predictive Controller for Reorientation and Disturbance Compensation in Underactuated Spacecraft

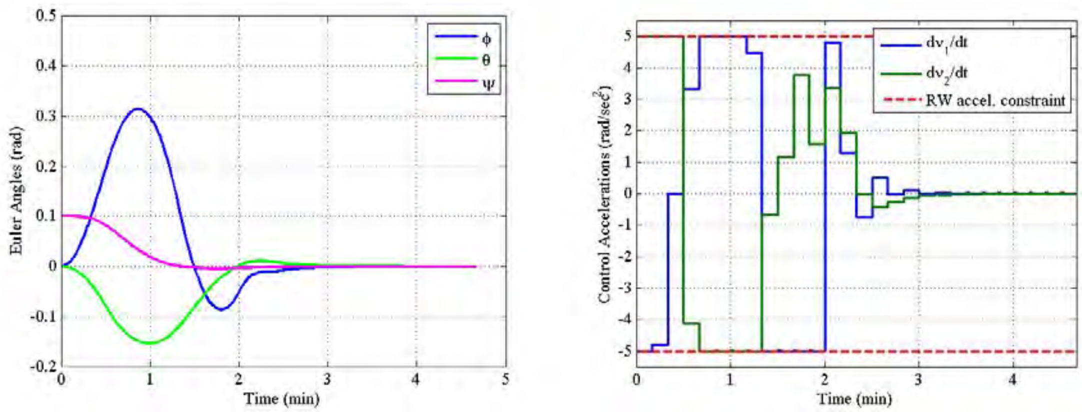
Figure 4 illustrates the use of the hybrid scheme to stabilize to the origin when there is an offset in the uncontrollable Euler angle. The initial conditions are such that the angular momentum is nonzero. As the fiber variable decreases to zero, the amplitude of the base variables decrease and they go to zero as well, converging to the correct orientation.



(a) **(b)**

Figure 4. Response of Spacecraft Controlled by two Internal Moments Produced by RWs and Produced by the Hybrid Controller to (a) Euler Angles, (e) Excitation Magnitude

Figure 5 demonstrates the MPC solution when applied to the reduced attitude equations. The advantage to MPC is that it allows for constraint enforcement. Therefore, constraints are added to prevent zero crossings of RW speeds and limits of RW accelerations. In this simulation, there is 5 rad/sec^{2}limit of RW acceleration. Despite the constraints, the controller is still able to guide the spacecraft to the desired attitude.



(a) **(b)**

Figure 5. MPC Response of Spacecraft Controlled by two Internal Moments Produced by RWs to (a) Euler Angles, (e) Control Accelerations

5.0 CONCLUSION

This work demonstrated several approaches to the attitude control of the underactuated spacecraft. The first method utilizes SRP to regain linear controllability of an underactuated spacecraft. With linear controllability restored, conventional control techniques such as LQ control can be applied. The second method demonstrates the ability to construct sets of initial conditions for which if a spacecraft starts within, it will satisfy pointing and state constraints regardless of the disturbance torque exerted on it. The third method demonstrates the use of hybrid and MPC schemes for the underactuated spacecraft attitude control. Both approaches deal with nonzero angular momentum, which is the case usually not considered in the current literature, yet has important consequences with regards to the operational life of the spacecraft. Both controllers are able to successfully guide to the attitude equilibrium.

Future work will be done on the development of the drift-sets approach. Constructing these sets can be computationally intensive, so the use of other methods such as those based on zonotopes is of great interest. In addition, utilizing these drift sets as a means to construct a control law is the next step of the evolution of the work. The MPC approach for the zero angular momentum case will also be studied more in depth, as a nonlinear controller that provides for rapid implementation will need to be developed.

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LIST OF SYMBOLS, ABBREVIATIONS, AND ACRONYMS

FUSE: Far Ultraviolet Spectroscopic Explorer

LQ. Linear Quadratic

RW: Reaction Wheels

SRP: Solar Radiation Pressure

MPC: Model Predictive Cntroller

ϕ: Roll Euler angle

θ: Pitch Euler angle

ψ: Yaw Euler angle

Θ: Vector of stacked Euler angles

$M(\Theta)$: Euler angles kinematics matrix

ω: Angular velocity

J: Inertia matrix of the spacecraft bus

J_w: Inertia of the RW about the spin axis

W: Orientation of the RW spin axes with respect to the spacecraft bus

ν: RW speeds

M_{ext}: External moments acting on the spacecraft

$\mathcal{O}_{BG}$: Direction cosine matrix from the inertial frame to the spacecraft frame

H: Inertial angular momentum vector

P_j: SRP pressure exerted on panel j

α: SRP constant

β: SRP constant

$\hat{u}_{nj}$: Normal to panel j point outward

$\hat{u}_s$: Unit vector from the center-of-mass to the sun

τ_j: SRP torque exerted by panel j

r_{jO}: Vector from center-of-mass to the center of panel of j

r_{CO}: Vector from the center-of-pressure to the center-of-mass

A_j: Area of panel j

$x(k)$: State at time instant k

$u(k)$: Control at time instant k

$w(k)$: Bounded disturbance at time instant k

A: Discrete state matrix

B: Discrete control matrix

$\overline{U}$: Set of acceptable control

$\overline{W}$: Bound on disturbance

$\overline{H}_i, \overline{h}_i$: Linear equability definitions

n: Number of linear equalities

O_k: Set of all initial conditions for which, if the spacecraft starts with, satisfies constraints at time k

Φ_{sun}: Solar Flux

C_{diff} : Diffusion coefficient

c : Speed of light

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